Flow on the symmetry plane of a total cavo-pulmonary connection

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Abstract

The flow inside a total cavo-pulmonary connection, a bypass operation of the right heart adopted in the presence of congenital malformation, is here studied for a specific geometry which has been recently introduced in clinics. The analysis has been performed by preliminary experimental observation and a novel Navier–Stokes formulation on the symmetry plane. This method, once some basic hypotheses are verified, allows to reproduce the flow on the symmetry plane of a three-dimensional field by using an extension of the two-dimensional approach. The analysis has confirmed the existence of a central vortex showing that it is not a real vortex (i.e. a place with accumulation of vorticity) but, rather, a weakly dissipative recirculating zone. It is surrounded by a shear layer that becomes spontaneously unsteady at moderately high Reynolds number. The topological changes and energy dissipation have been analysed in both cases of unbalanced and of balanced pulmonary artery and caval flows. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Three-dimensional modelling of fluid flows is generally necessary to uncover the possible phenomena that the motion may undergo because of geometry complexity or because of flow stability at increasing Reynolds number. Three-dimensional numerical simulations of the flow under realistic conditions are now becoming increasingly feasible even though they still remain as a challenging application of computational fluid mechanics. In several problems, the actual resolution which is necessary to reproduce the details of the flow field, especially when it is an unsteady one, is still at the limit of or beyond the present computational capabilities. Some Navier–Stokes solvers include methods to adjust numerical errors occurring locally when the resolution does not become uniformly adequate. These do not influence the flow field unless in some cases, usually associated with vorticity unsteadiness, when significant quantities are dependent on details of the flow which may not be accurately described.

On the other side there are some merits that can be put forward about the simpler two-dimensional formulations: the accuracy that can be obtained in two-dimensional simulations is extremely high; the generation of a finite element mesh and the numerical solution is very rapid so that the whole process from a geometry datum to a solution can be completed within minutes. In addition, the immediate interpretation of plane-defined quantities suggests the use of two-dimensional diagnostics and makes use of a two-dimensional approach actually feasible even for application in clinical practice. It is therefore natural to seek a two-dimensional formulation in applied problems. The main point in a two-dimensional formulation relies on the fact that the plane flows must represent a plane of the actual three-dimensional problem, and this is not often possible, although in some cases, it would represent an acceptable approximation of it.

The symmetry plane of a three-dimensional flow is a primary candidate to formulate a two-dimensional problem, at least when it can be assumed that the three-dimensional flow does not undergo a symmetry breaking. A symmetry plane formulation has been previously introduced for the boundary-layer flow (Ersoy and Walker, 1987; Pedrizzetti, 1992). However, the normal two-dimensional approximation produces fundamental errors even for representing a symmetry...
plane. The main point is that a two-dimensional flow is based on the conservation of mass and momentum on such a plane, whereas a three-dimensional flow does not require such conservations on the symmetry plane. A simple example is given by a converging tube that from an inlet diameter \( D \) reduces to an outlet diameter \( D/2 \), while the mass conservation gives an outlet velocity 4 times larger than the inlet one, if the mass was conserved on the symmetry plane, then the resulting outlet velocity would be just equal to twice the inlet one. As a result, a two-dimensional formulation is fundamentally incorrect even for the symmetry plane modelling.

In this work, we introduce a symmetry plane formulation of the three-dimensional incompressible Navier–Stokes equations which guarantees the conservation of mass and momentum in the corresponding three-dimensional flow. Such a formulation is then applied, at the completion of experimental measurements, to investigate the fluid dynamic behaviour for a case of total cavo-pulmonary connection (TCPC). The experimental flow was quantitatively studied with a glassblown model of the TCPC and the particle image velocimetry (PIV) technique, by means of the cross-correlation of single-exposure, consecutive images of the seeded flow.

The partial or total shunting of the right heart is often adopted to guarantee a sufficient perfusion of the lungs in the presence of certain congenital cardiac malformations. The TCPC considered here consists of the anastomosis of the two venae cavae to the pulmonary arteries (PA); the superior vena cava (SVC) is directly connected to the PA, whereas an extracardiac tunnel made of synthetic material (Dacron) is interposed between the inferior vena cava (IVC) and the pulmonary artery. In such a configuration, the flow is driven by the low pressure of the venous pathway which must be used as efficiently as possible, minimising the possible energy losses at the vessels’ connections.

The specific geometry of the TCPC considered here has been proposed at the Paediatric Hospital “Bambino Gesù” in Rome and it has proven to be successful in preliminary follow-up assessments (Amodeo et al., 1997). A glass-blown replica, and a computational domain, have been realised on the basis of NMR anatomical data. The geometry is characterised by a 6 mm, offset between the anastomoses of the venae cavae with the PAs, in order to provide a smooth partition of the flow with respect to previous realisations (Kim et al., 1995; Sharma et al., 1996).

It has been observed experimentally (Grigioni et al., 2000a,b) that this geometry creates a recirculation of the flow (a stagnation zone) in the region where the VCs connect to the PAs. Due to the crucial role of energy losses in such a procedure, it is important to verify the possibly beneficial role of this vortex. Physiological caval flows are considered here in correspondence to the balanced PA and to a higher arterial impedance due to a stenosed PA. Flow characteristics and pressure losses are discussed in a steady flow regime.

The role of the pulsatility in the TCPC is still a matter of debate: the flow due to the venous return has a reduced pulsatility (close to zero), and some authors prefer to neglect its importance with respect to the basic hydrodynamics imposed by the average flow rate (Senzaki et al., 1994; Kim et al., 1995). A steady entry flow is thus considered here; however the instability of the steady regime has been found at higher but still physiological values of the Reynolds number. The characteristics of this spontaneous fluid pulsatility are presented.

A TCPC similar geometry has already been studied (Dubini and de Leval, 1996) with a three-dimensional model, where symmetry plane hypothesis was used.

2. Methods

2.1. Symmetry plane formulation of the Navier–Stokes equations

Assume a Cartesian system of coordinates \( \{x, y, z\} \) and assume \( z = 0 \) as a symmetry plane of a domain bounded from above and below at \( z = \pm h(x, y) \).

On the symmetry plane, \( z = 0 \), the components of velocity and vorticity fields are odd or even functions of \( z \): the in-plane components of velocity are even functions, whereas the normal component is odd:

\[
\frac{\partial u_x}{\partial z} = 0, \quad \frac{\partial u_y}{\partial z} = 0, \quad u_z = 0; \tag{1}
\]

the opposite occurs for the vorticity components:

\[
\omega_x = \frac{\partial u_y}{\partial y} - \frac{\partial u_z}{\partial z} = 0, \quad \omega_y = \frac{\partial u_z}{\partial x} - \frac{\partial u_x}{\partial z} = 0, \quad \omega_z = 0 \tag{2}
\]

and we shall indicate with \( \omega \) the only non-zero component of vorticity \( \omega = \omega_z = \hat{\partial}u_x/\hat{\partial}y - \hat{\partial}u_y/\hat{\partial}x \).

Near the symmetry plane, we can express the velocity and vorticity fields by Taylor expansions including conditions (1 and 2):

\[
u_x(x, y, z) = u_x(x, y, 0) + O(z^2), \quad u_y(x, y, z) = u_y(x, y, 0) + O(z^2), \quad u_z(x, y, z) = g(x, y)z + O(z^2), \quad \omega(x, y, z) = \omega(x, y, 0) + O(z^2), \tag{3}
\]

where, for simplicity, we have introduced the function \( g(x, y) = (\hat{\partial}u_z/\hat{\partial}z)|_0 \) for the local gradient of normal velocity which represents the flow entering into or exiting from the symmetry plane.
We write down the equation of motion near the symmetry plane, expressing all variables in Taylor series (3), and retaining the leading terms in \( z \); in doing this, we consider a vorticity-streamfunction formulation of the Navier–Stokes equations. In what follows, variable names represent their values on the symmetry plane, i.e., \( z = 0 \). The only non-zero vorticity equation on the symmetry plane is

\[
\frac{\partial \omega}{\partial t} + u_x \frac{\partial \omega}{\partial x} + u_y \frac{\partial \omega}{\partial y} = \nu \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2} \right),
\]

(4)

where the appearance of a stretching term \( g \omega \) otherwise not present in purely two-dimensional flows is noticeable, and \( \nu \) is the kinematic viscosity. The three-dimensional continuity equation can be written as

\[
\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = -g,
\]

(5)

which shows that the field \( g \) represents the divergence of the velocity field on the symmetry plane.

Symmetry plane velocity can be expressed in general as the sum of an irrotational contribution plus a divergence-free one (Batchelor, 1967):

\[
\begin{align*}
u_x(x, y) &= \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}, \\
u_y(x, y) &= \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x},
\end{align*}
\]

(6)

where the streamfunction is related to vorticity in the standard way

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega
\]

(7)

and the potential from three-dimensional continuity (5):

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -g.
\]

(8)

Up to this point, we have given a mathematical formulation of the problem near the symmetry plane without introducing any additional hypothesis. However, the system of equations is not a closed one and additional relations between the variables are required to complete formulation (4)–(8). The problem can be closed by a relation of \( g \) and of \( \partial^2 \omega/\partial z^2 \) in terms of symmetry plane variables.

The value of \( g \) represents the flow exiting (\( g > 0 \)) or entering (\( g < 0 \)) into the symmetry plane from the third, \( z \), dimension. The flow is bounded from above and below at \( z = \pm h(x, y) \) and the conservation of mass over the fluid column gives the integrated continuity equation

\[
\frac{\partial}{\partial x}(h U_x) + \frac{\partial}{\partial y}(h U_y) = 0,
\]

(9)

where \( U_x \) and \( U_y \) are the velocity components averaged over the range of \( z \) between \( \pm h(x, y) \). Let us introduce the basic hypothesis that the symmetry plane velocity is proportional to an average velocity (or locally proportional with the proportionality constant \( \kappa \) slowly varying in \( x, y \)):

\[
u(x, y) = \kappa U(x, y).
\]

(10)

Assumption (10) essentially means that the velocity profile along the vertical is approximately self-similar in all points of the domain (or slowly changing); it is violated in the presence of significant three-dimensional phenomena, like, for example, important swirling flow or boundary-layer separation at the upper and lower walls, which cannot be reproduced by the conservation of momentum (4) near the symmetry plane only.

Substitution of hypothesis (10) into Eq. (9) gives an additional conservation law for the symmetry plane velocity

\[
\frac{\partial}{\partial x}(h u_x) + \frac{\partial}{\partial y}(h u_y) = 0,
\]

(11)

which, with the aid of continuity (5), gives the variable \( g \), representing the three-dimensional flux due to the confinement, as a function of symmetry plane variables

\[
g = \frac{1}{h} \left( \frac{\partial h}{\partial x} u_x + \frac{\partial h}{\partial y} u_y \right).
\]

(12)

Substitution of the velocity expression in terms of potential and streamfunction (6) inside (11), and putting the latter in the rhs of (11), gives the elliptic equation for the potential:

\[
\frac{\partial}{\partial x} \left( h \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( h \frac{\partial \phi}{\partial y} \right) = \frac{\partial h}{\partial x} \frac{\partial \psi}{\partial y} - \frac{\partial h}{\partial y} \frac{\partial \psi}{\partial x}.
\]

(13)

An additional closure is necessary to express the viscous term \( \partial^2 \omega/\partial z^2 \) near \( z = 0 \); however, its relevance being less structural, the assumption \( \partial^2 \omega/\partial z^2 = 0 \) is taken here, which is consistent with Poiseuille flow and with the general fact that such a viscous term is not relevant far from the walls at \( z = \pm h(x, y) \). Alternatively, a relation between this and the other viscous terms can be assumed.

If required, the in-plane pressure can be evaluated a posteriori by taking the divergence of the three-dimensional Navier–Stokes equation to give, on the symmetry plane

\[
\frac{\partial^2 \nu}{\partial x^2} + \frac{\partial^2 \nu}{\partial y^2} = -\left( \frac{\partial \nu_x}{\partial x} \right)^2 - 2 \left( \frac{\partial \nu_x}{\partial x} \right) \left( \frac{\partial \nu_y}{\partial y} \right) \left( \frac{\partial \nu_x}{\partial y} \right) - \left( \frac{\partial \nu_y}{\partial x} \right)^2 - \frac{\partial^2 \nu}{\partial z^2},
\]

(14)

which presents two additional terms with respect to the two-dimensional case. The last term \( \partial^2 \nu/\partial z^2 \) is the one responsible for secondary, and helical, flow which are not reproduced in the symmetry plane approximation; it is therefore neglected in the present study. However, it has been verified to be really negligible in this application by computing this term in one case by the use of the
momentum equation for $g$:

$$\frac{\partial y}{\partial t} + u \frac{\partial y}{\partial x} + u \frac{\partial y}{\partial y} = -\frac{\partial^2 p}{\partial x^2} + v \left( \frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} \right) + \nu \frac{\partial^3 y}{\partial x^3} |_0,$$

(15)

where the last viscous term, that cannot be written in terms of symmetry plane variables, has been neglected.

2.2. Numerical method of solution

The numerical method is analogous to the $\omega$-$\psi$ formulation to solve pure plane flow (Roache, 1998; Pedrizzetti, 1996; Zovatto and Pedrizzetti, 2001) and it contains just an additional step for the solution of the potential $\phi$ with respect to the standard numerical method in the $\omega$-$\psi$ formulation.

The space domain has been discretised with linear triangular finite elements and equations have been rewritten on the finite element mesh using a Galerkin residual procedure (Zienkiewicz, 1977), resulting in a second-order accuracy in space.

The vorticity equation (4) can be written in the matrix form as

$$\mathbf{B} \frac{d\omega}{dt} + \mathbf{C}(\psi, \phi, g) \omega + \frac{1}{Re} h\phi = 0,$$

(16)

where the variables represent the vector of their values at the nodes of each element; $\mathbf{B}$ is the element mass matrix, $\mathbf{K}$ is the stiffness matrix which corresponds to the discretised Laplace operator, and $\mathbf{C}(\psi, \phi, g)$ represents the non-linear operator applied to vorticity. Eq. (16) is discretised in time with a second-order scheme fully implicit for the viscous term, the vorticity field at all the internal points is thus advanced in time with

$$\frac{(3\omega^{(n+1)} - 4\omega^{(n)} + \omega^{(n-1)})}{2\Delta t} + \frac{1}{Re} h\phi^{(n+1)} = 0,$$

(17)

where the superscript $(n)$ represents the $n$th time step, while the superscript $(*)$ corresponds to the extrapolation $f(*) = f^{(n)} - f^{(n-1)}$.

The Poisson equation (7), discretised as

$$\mathbf{K}\psi = \mathbf{B}\omega,$$

(18)

allows us to determine the value of $\psi$ everywhere once the vorticity is known at all points internal to the domain. Boundary conditions of Neumann type are taken at the inlet and outlets while Dirichlet conditions are assumed at the rigid walls representing the flowrate in the absence of three-dimensional effects.

Afterwards, the potential elliptic equation (13) is solved with impermeability, Neumann, boundary conditions at the rigid walls. It must be noticed that the irrotational boundary condition (impermeability) can be arbitrary for $\psi$ and $\phi$ because these are defined independently of a possible additional incompressible and irrotational field (so no vorticity nor inflow) just to readjust the irrotational boundary conditions; for convenience, we can use the impermeability conditions either for $\psi$ or $\phi$. Discretisation of Eq. (13) with Neumann condition $\partial \phi / \partial n = 0$ at the rigid boundaries results in a system

$$h K \phi = F(h, \psi),$$

(19)

whose solutions give the potential $\phi$.

At the inlet and outlet sections, the geometry is uniform and a normal flow is assumed, $\partial \phi / \partial s = \partial \psi / \partial n = 0$. This gives an additional condition for $\phi$ which is fixed at an arbitrary (zero) value on one section while the condition of constancy to an unknown value is assembled into the matrix in (19), such a condition can be imposed at the same time with the condition of zero additional flow $\partial \phi / \partial n$ in the sections where the flow is given.

Finally, the value of vorticity at the rigid walls is computed by solving back Eq. (18) with the values of $\omega$ at the walls as unknowns, the no-slip condition is transformed in imposing a value to the normal derivative for the streamfunction, $\partial \psi / \partial n = \partial \phi / \partial s$ (where $s$ indicates the tangential direction), in the residual Galerkin formulation (Saiac et al., 1995; Zovatto and Pedrizzetti, 2001). This procedure gives the same order of accuracy for the calculation of the vorticity at the wall and at the internal points. The other quantities like $g$ and velocities are then computed explicitly by (6) and (11). This methodology has proven to be accurate both in steady and unsteady regimes.

The following numerical parameters have been used for the results of Section 3 below. The number of nodes employed for the discretisation ranges from about 18,000 to 23,000 (34,000-44,000 elements) depending on the Reynolds number; care has been used in refining the grid at the central zone where thin boundary layers are expected.

Preliminary grid refinement tests have been performed to verify that the mesh was sufficiently accurate to resolve all details; the use of a coarser mesh (obtained by dividing the number of elements by 4) gives the same gross features; however, it was unable to cleanly resolve some details near the walls and therefore to correctly evaluate the wall shear stress distribution. The timestep has been chosen to satisfy the convective stability condition and the diffusive accuracy. Every simulation in Section 3 has been impulsively started from the irrotational field, in all cases, the regime flow is reached after about 50–100 time units (defined below), the results reported below are given after 400 time units.
2.3. Preliminary test

The symmetry plane model introduced above is here applied to the simple case of a flow inside a converging circular duct as a preliminary verification of its consistency, and to exemplify the situation where it can be applied and discuss where it is not applicable because the basic hypothesis (10) is violated.

Three-dimensional converging flow involves the entry of flow into the symmetry plane: that is shown by \( g \) field, negative in the converging zone, where there is an increase in the specific mass into the symmetry plane. The symmetry plane flow for a case where the tube radius is halved is shown in Fig. 1 and compared with the two-dimensional case with the same flow rate at the entrance. The three-dimensional conservation of discharge is verified exactly on the symmetry plane and it is shown by the inlet and outlet velocity profiles: the symmetry plane shows an outflow discharge which is correctly twice the inlet one, while the two-dimensional model presents the same flowrate at the two ends.

The value of the \( g \) field, reported in the picture, shows the flow entering the symmetry plane in the converging part of the duct. The irrotational additional potential \( \phi \) is also reported, \( \phi \) values are regularly increasing towards the outlet where a constant additional velocity is established.

It is also necessary to comment on situations when the symmetry plane model is not expected to be applicable like it could be in the case of a diverging duct. In fact, hypothesis (10) states that the flow out above and below the plane of symmetry behaves on average in the same way as the symmetry plane flow. This essentially means the obvious fact that in the symmetry plane approximation, we cannot model phenomena occurring out of such a plane that does not have a signature on it. In the case of a diverging duct, we expect a boundary-layer separation and a recirculation region on the upper and lower walls that cannot be reproduced by the conservation of momentum (4) near the symmetry plane only. In this case, although three-dimensional conservation of mass is still forced, the hypothesis of a velocity that is proportional to the symmetry plane velocity is clearly violated by the appearance of an inversion of flow near the upper and lower walls. These situations cannot be approached by a symmetry plane approximation even in a first approximation and, like those cases where a symmetry plane flow is unstable and relevant helical flow establishes itself, must be studied by experimental or by three-dimensional computational studies.

2.4. Experimental method

A blown-glass replica of the TCPC, realised in accordance with NMR data relative to the previously operated paediatric patients, was inserted in a closed loop. As opposed to other geometries chosen for in vitro studies of the TCPC, the one considered here is derived directly from anatomical data. The geometry realised at the “Bambino Gesù” Paediatric Hospital in Rome has proven to be successful in follow-up assessments (Amodeo et al., 1997). The TCPC replica’s main geometric features are the following: IVC and SVC diameter 11.2 mm, main PA diameter 8.7 mm; the IVC axis at the anastomosis level, inclined towards the LPA, forms a 22° angle with the vertical direction, and the offset between the IVC and SVC anastomoses is 6 mm.

The flow was maintained steady by means of the fixed pressure difference between the VCs and the PAs, set by the position of the corresponding upstream (VC) and downstream (PA) reservoir. The system has been described in full detail in Grigioni et al. (2000b). The 6-mm offset between the anastomoses of the venae cavae with the PAs provides a smooth partition of the flow coming from the VCs, as confirmed by PIV investigation although the role of the observed central vortex is still unclear.

3. Results

3.1. General observations

The techniques outlined above are used to analyse the fluid-dynamics features of the flow inside the specific geometry of TCPC proposed. The geometry dimensions are given in Section 2.4, whereas the overall shape is shown in Fig. 2 (see also pictures below) where the photo of the central part of the glass replica is shown underlying the experimental data.

We consider here the response of the flow in this specific geometry to variations of the possible
parameters. Once the geometry is fixed, we can identify three dimensionless free parameters which define univocally the possible flow configurations. These are (i) the flow ripartition between the two venae cavae, i.e. the ratio of the discharge coming from the inferior and the superior venae cavae $r_{vc} = Q_{ivc}/Q_{svc}$; (ii) the flow ripartition between the two pulmonary arteries $r_{pa} = Q_{lpa}/Q_{rpa}$; and (iii) one appropriately defined Reynolds number, in what follows, we consider the mean Reynolds number of the two pulmonary arteries $Re = 2Q/\nu D_{pa}$, where $Q$ is the total discharge entering the system, $D_{pa}$ is the diameter of the pulmonary artery, and $\nu$ is the blood kinematic viscosity. In what follows, the dimensionless quantities are obtained by normalisation with the mass density $\rho$, the diameter $D_{pa}$, and the velocity scale $2Q/\pi D_{pa}^2$ (which is the mean between the RPA and LPA velocities).

In Fig. 2, the velocity vectors measured experimentally by the PIV technique are shown on the symmetry plane of the in vitro model. These are relative to a reference flow condition, at $Re = 700 (Q = 1.91/min)$, with a realistic caval flow ratio $r_{vc} = 6/4$, and balanced pulmonary flow $r_{pa} = 1$.

The central region of the phantom is shown, upstream of this (venae cavae), the flow is a Poiseuille uniform one and tends to uniform flow a few diameters downstream into the pulmonary arteries. A relevant central vortex is evident, it is due to the portion of flow coming from the inferior vena cava which must turn to enter the right pulmonary artery (RPA, the vessel at the left side of the picture). This “vortex” is more correctly a circulation zone because such a rotation is not necessarily associated with an accumulation of vorticity at its centre. From this figure, it is not clear if such a vortex plays a significant role in pressure losses or if it may even be beneficial as an efficient flow divider. However, its presence creates a series of attachment and separating points at the wall with subsequent anomalous distribution of wall shear stress.

The numerical solution in correspondence to the same parameters is reported in Fig. 3 where the velocity vectors (above) and the vorticity field (below) are reported. The uniform flow condition is imposed at the far ends, inlet/outlet, of the computational domain with the known value of the symmetry-plane flowrate as measured. The velocity vectors are drawn with the same scale used in Fig. 2, a moderate flow entering into the symmetry plane is found at the entrance of the pulmonary arteries. The numerical solution presents a central vortex analogous to that found experimentally; some differences in the geometry of the connection, more constrained in the experimental phantom, may alter the exact arrangements of the weak stagnating region.

The fully developed Poiseuille flow coming from the SVC enters smoothly into the RPA, accelerating during the curve and avoiding a boundary-layer separation. Differently, the flow from the IVC splashes on the facing wall because a part of its flow does not enter the LPA and must turn sharply to enter the farther RPA. The vorticity field evidences that the “vortex” is not a region with high vorticity; rather, it is a stagnating region bounded by the wall and by vorticity layers. An intense vorticity layer comes from the boundary layer developed
after the stagnation point, it elongates towards the RPA and divides the flow coming from the SVC and from the IVC.

3.2. Results at varying Re, instability of the steady regime

The previous flow picture remains analogous although with sharper vorticity structures when the

Reynolds number is increased. The vorticity field (above) and the dimensionless pressure field (below) are shown in Fig. 4 at \( Re = 1100 \) and same flow ripartition. It is evident that the boundary layer at the IVC splash becomes thinner and presents higher vorticity values; the separated shear layer is significantly more elongated towards the RPA. The main features in the pressure field, that does not present significant differences, are the high pressure values about the
stagnating point of the splashing zone and a significant pressure decrease at the entrance of the pulmonary arteries. The pressure drop at the PA entrance is not an energy loss, rather a transformation into kinetic energy. The mean flow accelerates entering the PA (area ratio 1.66) with respect to the VC velocity value, and kinetic energy increases by almost three times.

The separated vorticity layer further elongates with increasing Reynolds number. At values higher than \( Re = 1100 \), the tail of the vorticity begins to fluctuate settling in a time-periodic regime although a steady flow is imposed at the boundaries. The unsteady vorticity field is shown in Fig. 5 at \( Re = 1600 \) in correspondence to six instants during the periodic cycle. The unsteadiness is still limited to the negative shear layer which undergoes a sort of Kelvin–Helmholtz instability: the elongated vortex layer locally rolls up, at the tail, until it detaches in a small vortex that enters the RPA and the cycle is restarted. The Strouhal number of this spontaneous unsteadiness is estimated as 0.137, therefore, the frequency is 3–4 times larger than the natural heartbeat. This is a value comparable with the frequency of phenomena occurring during a heartbeat, therefore, an interference cannot be excluded a priori.

The dependences of pressure and energy losses on the Reynolds number are evaluated for a physiologically realistic range by evaluating the energy losses and pressure losses coefficients \( K_e \) and \( K_p \):

\[
K_e = \frac{(p_{ivc} + (\rho/2)V_{ivc}^2 + p_{svc} + (\rho/2)V_{svc}^2) - (p_{rpa} + (\rho/2)V_{rpa}^2) + p_{lpa} + (\rho/2)V_{lpa}^2)}{[(\rho/2)(V_{ivc}^2 + V_{svc}^2)]},
\]

(20)

\[
K_p = K_e + \frac{(V_{rpa}^2 + V_{lpa}^2) - (V_{ivc}^2 + V_{svc}^2)}{(V_{ivc}^2 + V_{svc}^2)},
\]

(21)

Fig. 5. Instantaneous vorticity contours for the unsteady regime at \( t = (1 \text{ to } 6) \times T/6 \), (a-f, \( T \) is the estimated period); \( Re = 1600 \). Levels from \( \pm 0.3 \) step 0.6: positive levels (gray lines), negative levels (black lines).
where the control sections are here taken at the ends of the computational domain which presents vessels approximately 4 units (the PA diameter) long. As reference values the loss coefficients are also estimated under the assumption of Poiseuille flow along the rectilinear parts of the vessels, this gives an energy loss (per unit volume)

\[ \Delta E_i = \frac{128}{\pi} \rho \nu \frac{Q_i L_i}{D_i^4} \]  

(22)

where \( Q_i, L_i, D_i \) indicate the discharge, the length, and the diameter of the \( i \)th segment (\( i = \text{ivc, svc, rpa, lpa} \)). Inserting the estimate (22) into (20), and assuming \( L_i = L \), we obtain

\[ K_e = \left( 1 + \left( \frac{D_{\text{vc}}}{D_{\text{pa}}} \right)^4 \frac{(1 + r_{\text{vc}})^2}{1 + r_{\text{vc}}^2} \right) \frac{L}{D_{\text{pa}} \Re} \]  

(23)

which in the present case would give (taking \( L \sim 4D_{\text{pa}} \)) \( K_e \sim 1000/\Re \), the pressure loss coefficient differs for a Reynolds-independent value

\[ K_p = K_e - 1 + \left( \frac{D_{\text{vc}}}{D_{\text{pa}}} \right)^4 \frac{(1 + r_{\text{vc}})^2}{1 + r_{\text{vc}}^2} \frac{1 + r_{\text{pa}}^2}{(1 + r_{\text{pa}})^2} \]  

(24)

here \( \approx 1.65 \).

The loss coefficients are reported in Fig. 6 with \( \Re \). The energy loss presents a behaviour that is intermediate between \( \Re^{-1} \), as would be expected in a Poiseuille regime (23), and \( \Re^{-1/2} \), as would be expected in a laminar entry tube flow. In fact, the two venae cavae are presumably in a Poiseuille regime, whereas the two pulmonary arteries are more similar to an entry flow with a growing boundary layer, the whole system being thus in an intermediate behaviour. The pressure losses are substantially higher than energy losses, and they are well described by an approximate energy losses law once the dominating contribution of pressure to kinetic transformation (24) is accounted for. It must be mentioned that the unsteadiness discussed above plays no role in energy and pressure losses, up to this \( \Re \), which are almost constant during a cycle and fluctuations are contained within the circles of the graphs in Fig. 6.

In conclusion, the major pressure difference is given by transformation of kinetic energy due to the variation of diameter from the venae cavae to the pulmonary artery; this is a phenomenon that cannot be captured by a two-dimensional approximation being given by three-dimensional mass conservation; the central vortex does not appear to play a relevant role in losses, rather it is important for the birth of vorticity unsteadiness.

### 3.3. The fluid dynamics at different flow ripartition

The fluid dynamic behaviour of the system has now been analysed at \( \Re = 700 \) at varying values of the flow ripartition, both \( r_{\text{vc}} \) and \( r_{\text{pa}} \) have been varied for the ratios \( 3/7, 4/6, 5/5, 6/4, 7/3 \) for a total of \( 5 \times 5 \) simulations.

The streamlines are reported in Fig. 7 for some values of the flow ripartition. In Fig. 7(a) and (b), for the regular \( r_{\text{vc}} = 6/4 \) ratio, the two extreme pulmonary flow divisions are shown. It is seen that the vortex changes
place from below to above depending on the flow division, in particular depending on whether the IVC flow must turn to the RPA (a) or if the SVC flow must enter into the LPA (b). In all the cases we can say that, in a first approximation, the central vortex has a relative strength which is proportional to the difference between the IVC and LPA flows (or equivalently between RPA and SVC)

$$\Gamma \propto \frac{Q_{ivc} - Q_{lpa}}{Q}$$

Thus, the pictures in Fig. 7 (as well as in Figs. 8 and 9) have relative vortex strengths which are $\Gamma = [0.3, -0.1, 0, -0.2, -0.4, -0.2]$, from (a) to (f), respec-

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Fig. 7. Streamlines for $Re = 700$ at varying flow ripartition.
tively; the negative sign means that the vortex is attached to the upper wall, while a positive vortex is attached to the lower wall as in Fig. 7(a) and like the case discussed in the previous sections. These vortex modifications are in agreement with the experimental observations (Grigioni et al., 2000a, b).

The flow topology also corresponds to a different arrangement of the vorticity shear layer and to the wall shear stress distribution at the wall. The vorticity fields are shown in Fig. 8(a–f) in correspondence to the same cases, in Fig. 7. In all cases, a splash of one vena cava, the IVC when $\Gamma > 0$ and the SVC for $\Gamma < 0$, on the facing
wall is found with the development of opposite sign vorticity layer at the wall that detaches and elongates around the circulating region.

Pressure distributions are reported in Fig. 9(a–f). The main features are in common among all flow ripartitions; these are a high pressure at the stagnating splashing point whose intensity is still related to $\Gamma$, and a pressure decrease at the entrance of each pulmonary artery. This decrease is particularly marked in the artery with the highest flow rate. Although the flow combines several viscous phenomena, the pressure distribution is essentially well represented by a Bernoulli balance, indicating that the actual energy dissipation has little importance in this district. The central circulation

![Fig. 9. Pressure field for $Re = 700$ at varying flow ripartition. Levels at step 0.02 from an arbitrary zero value, positive levels (black lines), negative levels (gray lines).](image-url)
plays a small role in dissipation, in fact, this is not a real vortex, with an associated pressure minimum, rather a weak circulation without vorticity at its centre. The significant vorticity layer is the only region where some viscous dissipation takes place; however, this is like a slight increase in the length of wall vorticity and not a structural, dominant, pressure drop.

The dependence of the energy losses with flow ripartition is presented in Fig. 10. At a fixed caval flow, the energy losses are weakly dependent on the pulmonary division, as would be in the case of Poiseuille flow (23). The dependence with caval flow shows an approximately symmetric shape with maximum $K_e$ at balanced pulmonary flow with a shape very similar to what is predicted, at a given $Re$, by the estimate (23). The non-symmetrical arrangement of the proposed geometry can be seen in the slight non-symmetry of Fig. 10.

4. Conclusions

A novel method for the simulation of the Navier–Stokes flow on the symmetry plane of a three-dimensional field has been introduced. This technique has been shown to be appropriate when a self-similarity in the velocity profile normal to the symmetry plane holds; thus, it cannot model rapidly diverging ducts or significant helical flows. However, in its range of applicability, it allows the use of rapid and accurate plane computations with three-dimensional conservation of mass and momentum.

This technique has been applied for the analysis of the fluid mechanics associated to a specific TCPC already adopted in the clinical practice (Amodeo et al., 1997). The analysis has been stimulated by the experimental observation of a central vortex at the connection between the two venae cavae and the pulmonary artery (Grigioni et al., 2000b). The numerical results have confirmed this observation and clarified the role of this vortex.

There is no real vortex at the connection; rather, a weak stagnating zone is found, without vorticity at its centre, surrounded by the wall and by a separated vorticity layer. In such a structure, the pressure does not show the typical drop present at the centre of a vortex, in this sense the vortex is a weakly dissipative, beneficial, flow division structure. On another side, the associated shear layer becomes unstable at a moderately high Reynolds number creating a spontaneous unsteadiness of a period of about four times smaller than normal heartbeat. The central circulation also creates a complex distribution of wall shear stress, with separating and stagnating zones, which may affect the quality of flow structure interaction. Results have been confirmed and characterised at a varying pulmonary and caval flow ripartition.
The limitation of a symmetry plane approximation does not allow to draw any definitive clinical conclusion. It gives a view of the phenomena at varying parameters and preliminary indications that must be verified by a three-dimensional approach for the cases of specific interest. An in vitro work focused on the unsteady phenomena detected here is in progress, a computational analysis is also possible although it is still challenging for the accuracy needed to resolve wall and shear layers which are the primary factors responsible for this unsteadiness.

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References


